On Routing Policies for Synchronized Queues

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Plan

1. Motivation
2. Model
3. Preliminary Results
4. Numerical Results
5. Conclusions
6. References

Motivation

- Cloud computing

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- Large network of parallel servers

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- Cost of information, single centralized queue is not scalable

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- All pieces must start service at the same time, FCFS

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Cost of Information vs. Service Quality?

We analize the **expected waiting time** in the 3 cases.

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Analyzed by Green [G80] with service time of pieces $S \sim \text{Exp}(\mu)$.

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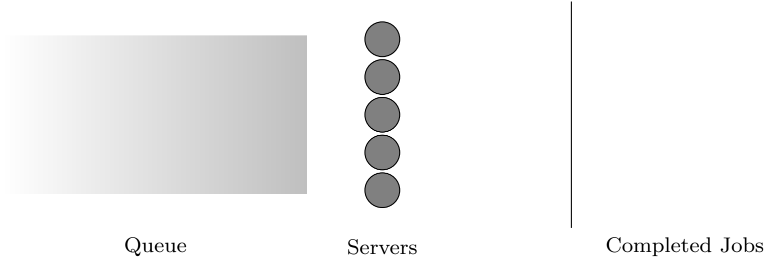

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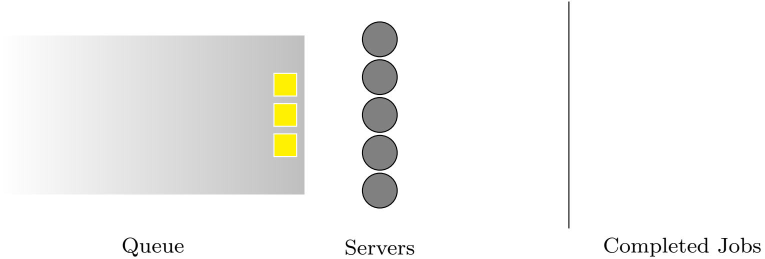

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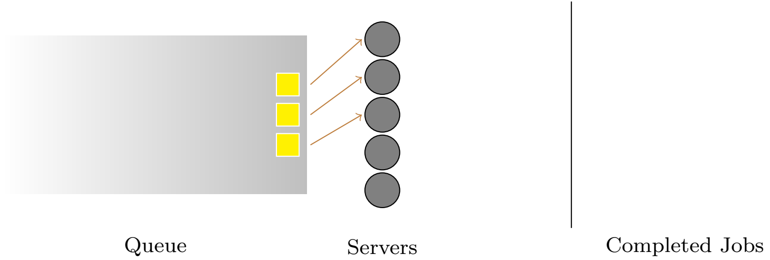

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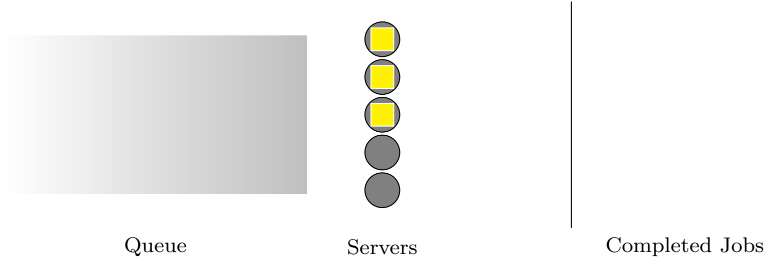

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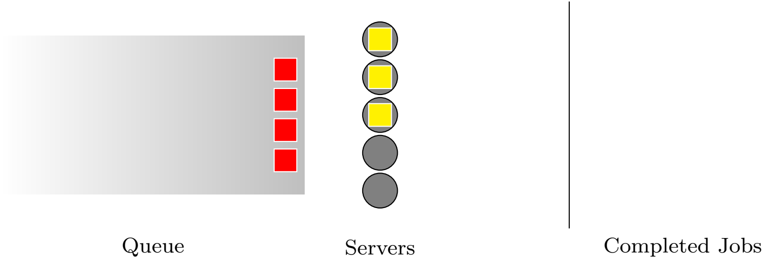

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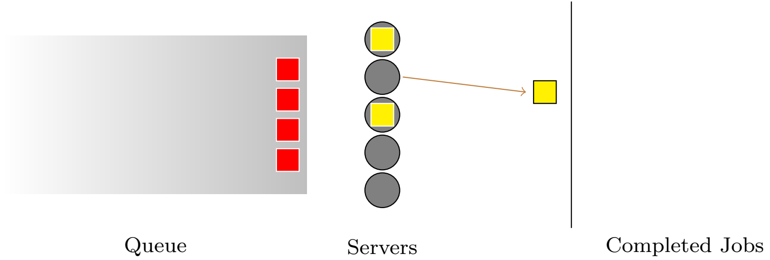

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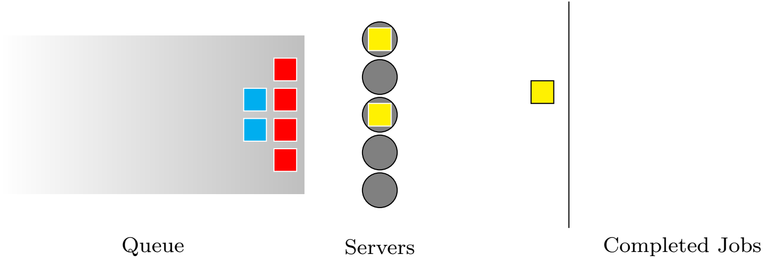

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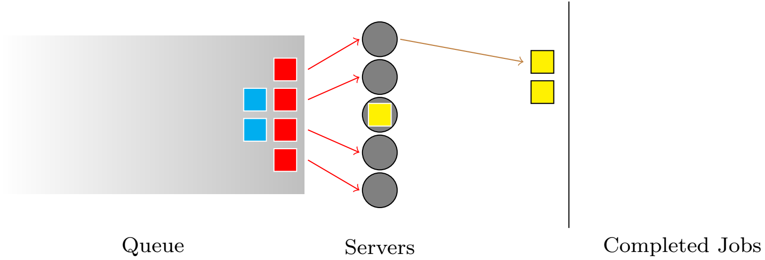

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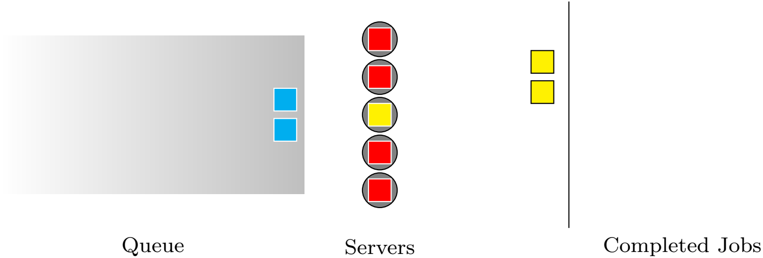

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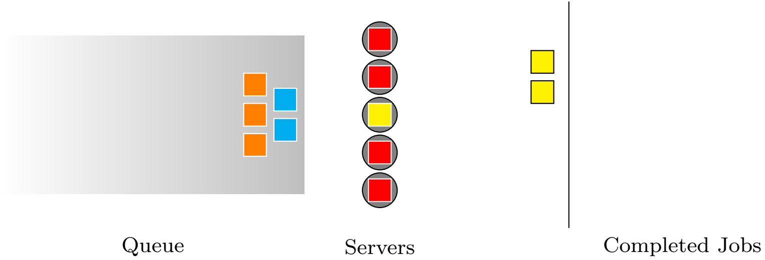

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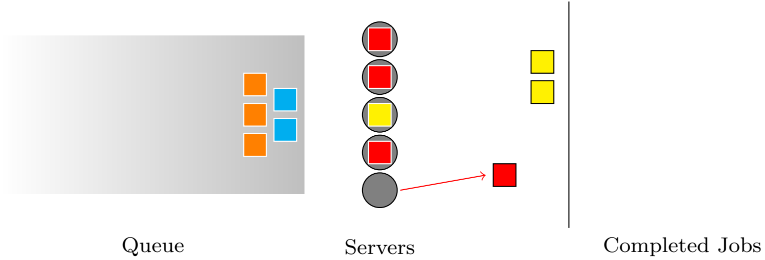

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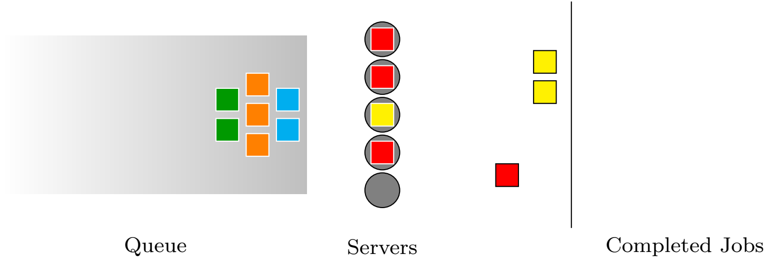

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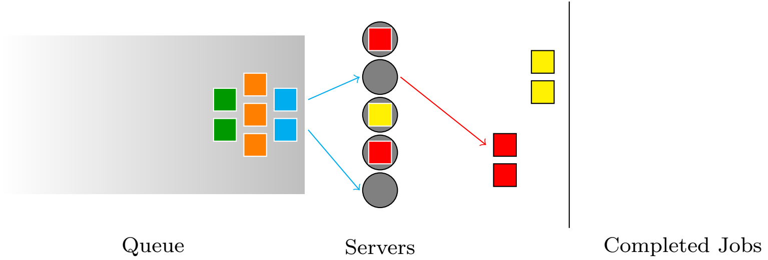

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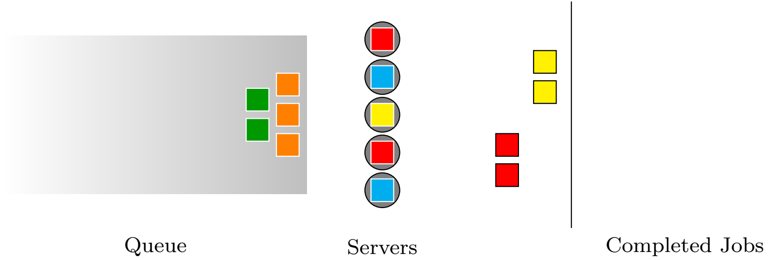

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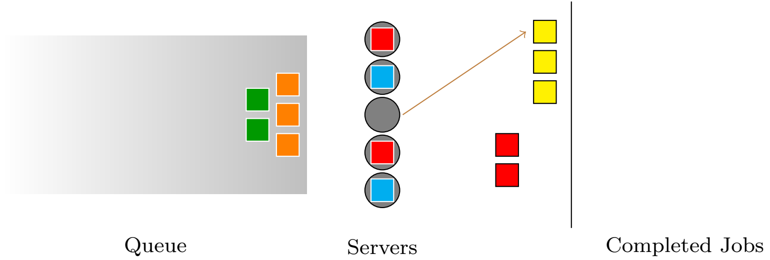

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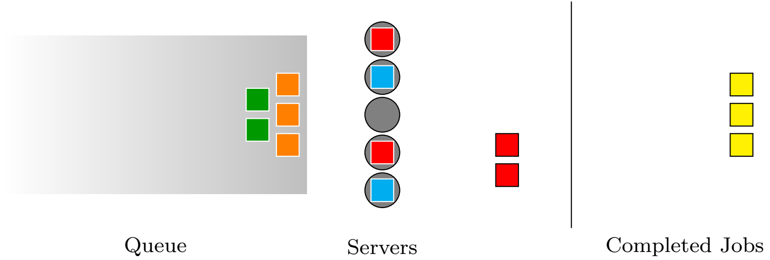

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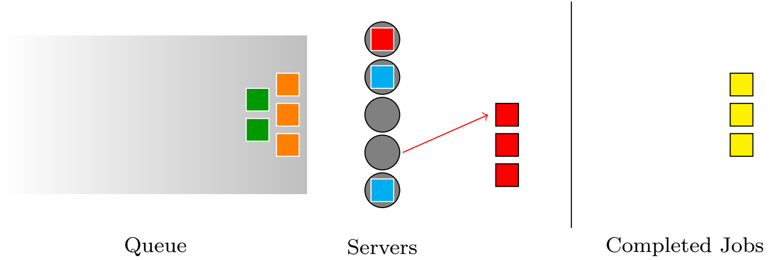

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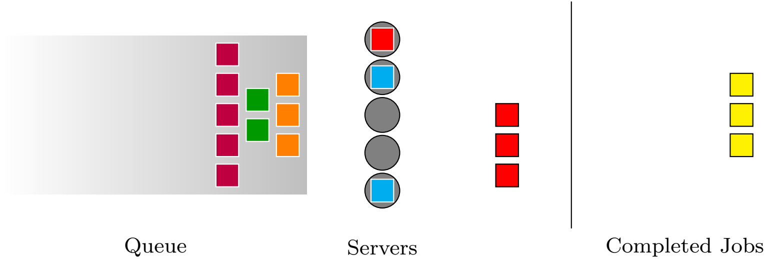

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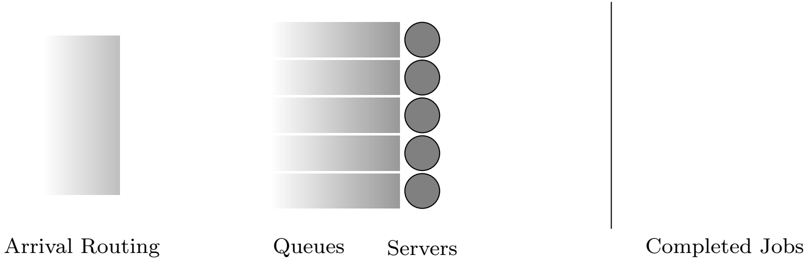

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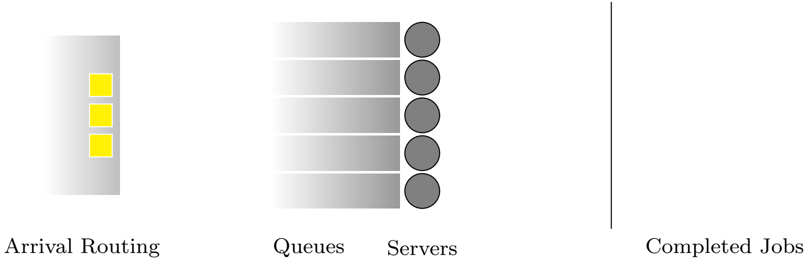

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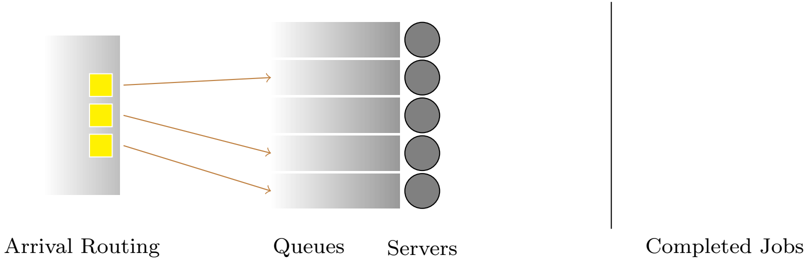

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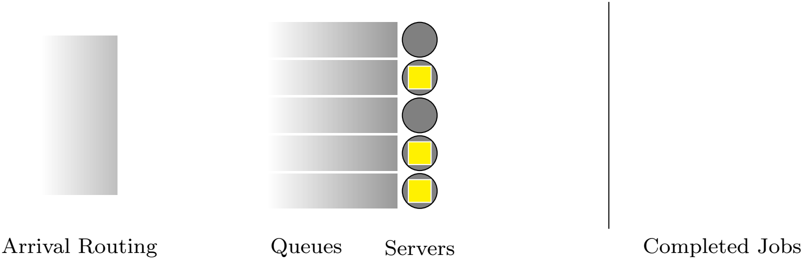

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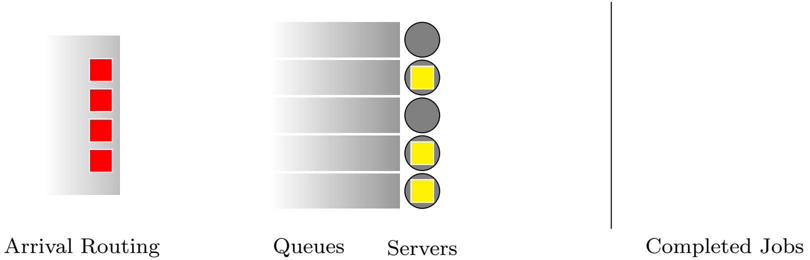

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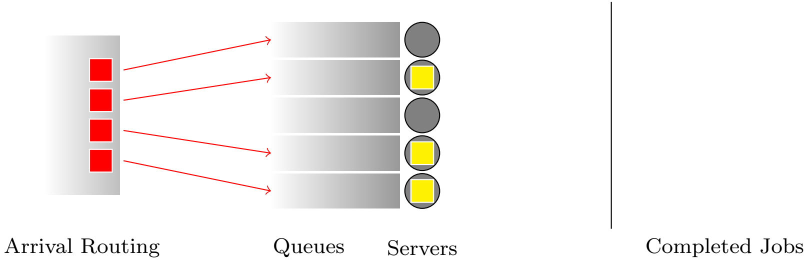

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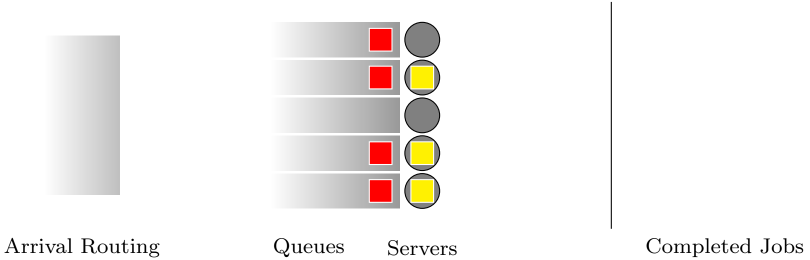

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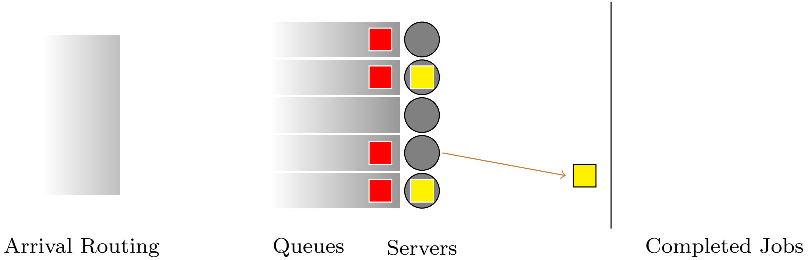

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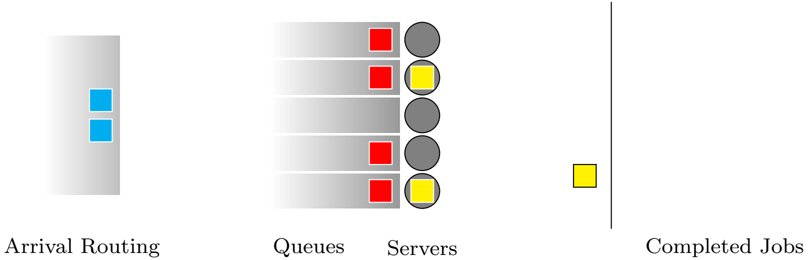

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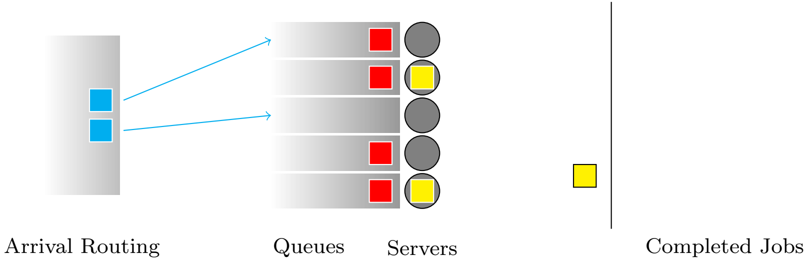

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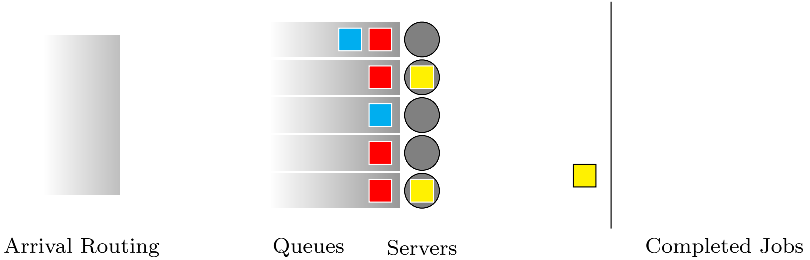

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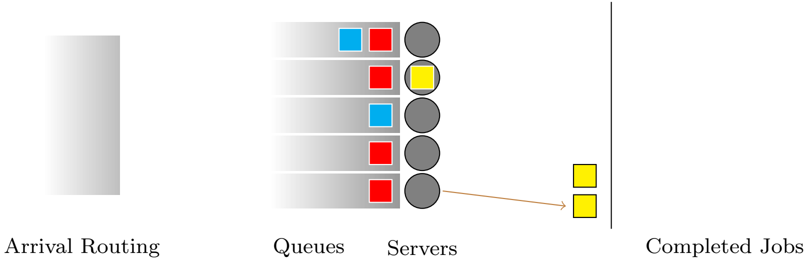

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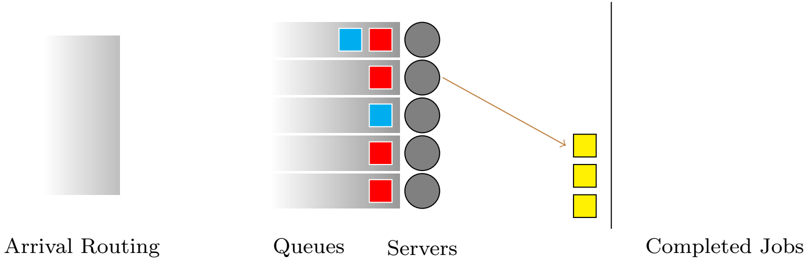

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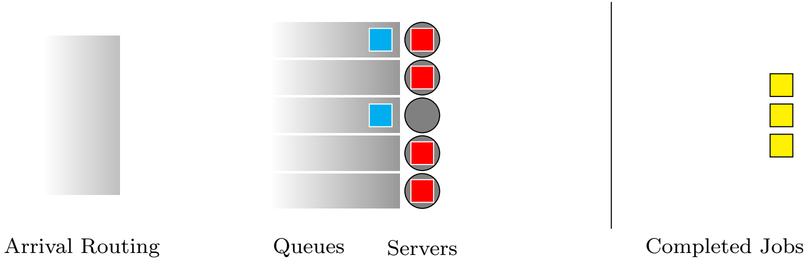

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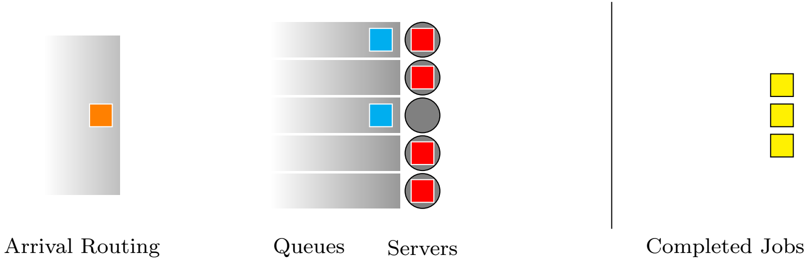

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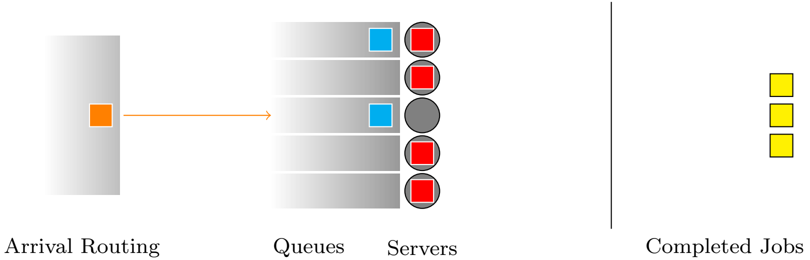

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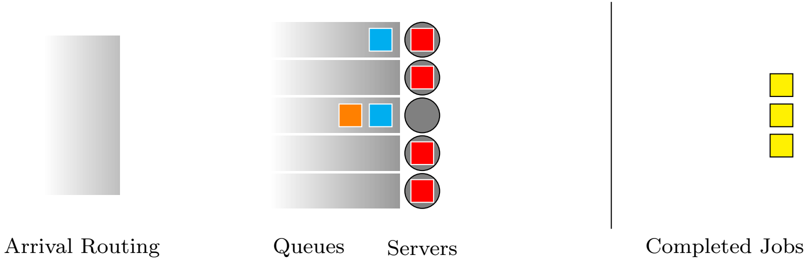

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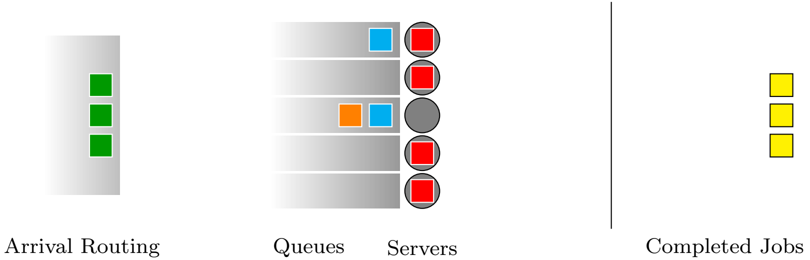

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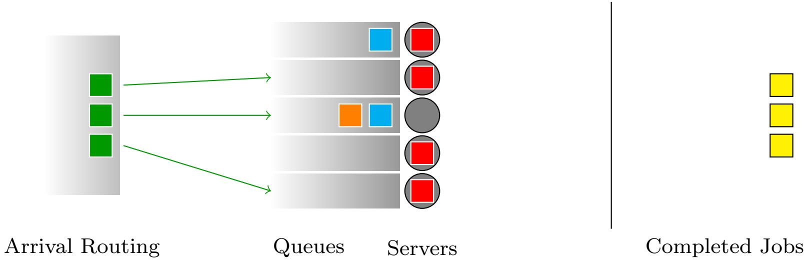

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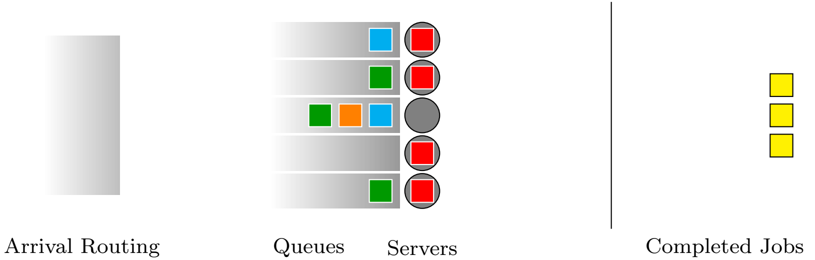

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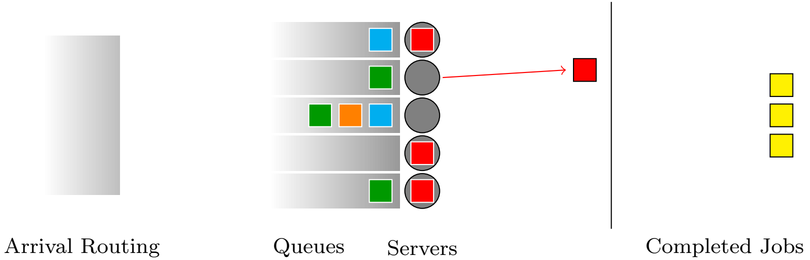

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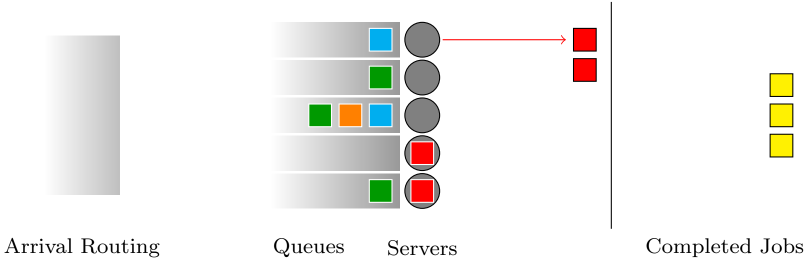

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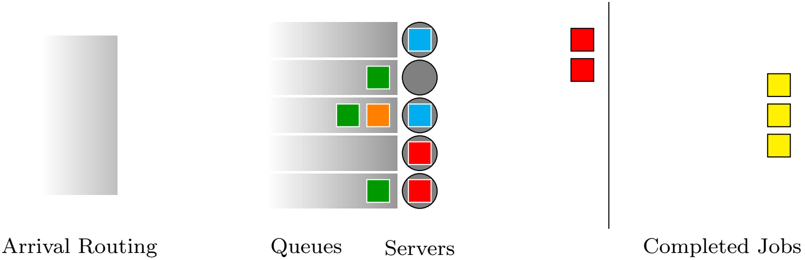

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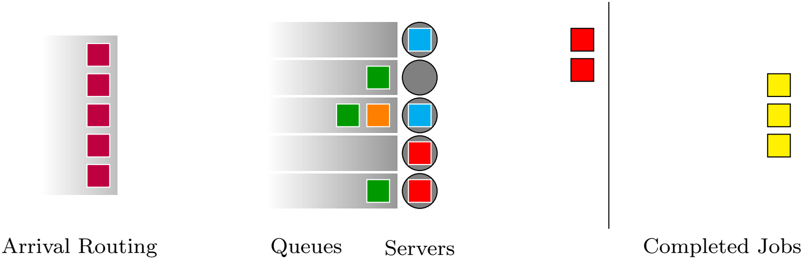

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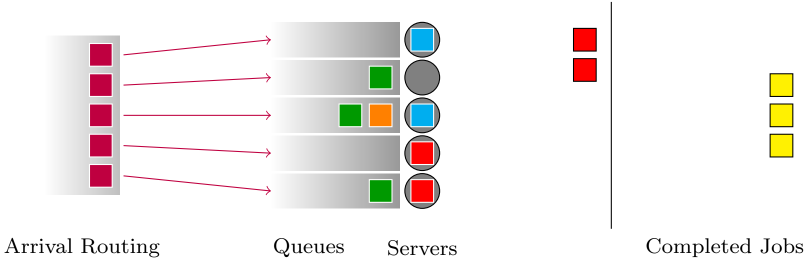

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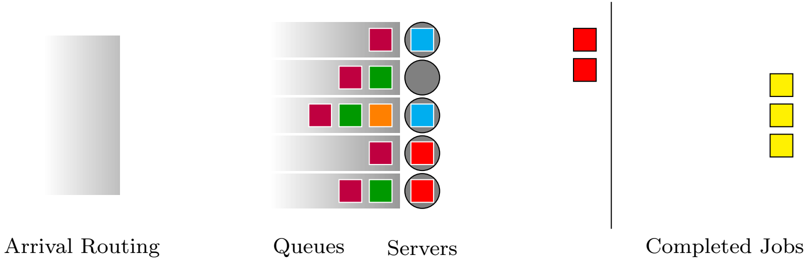

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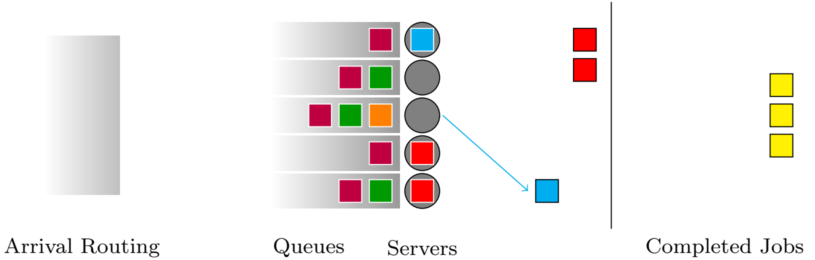

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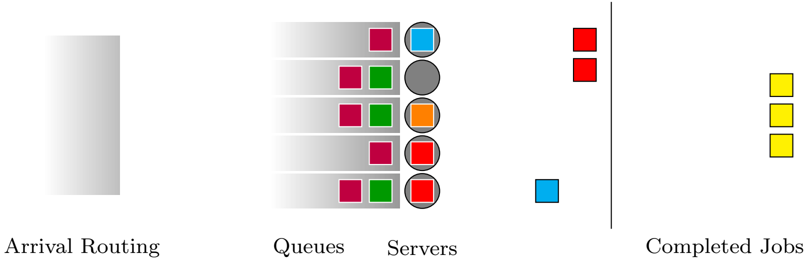

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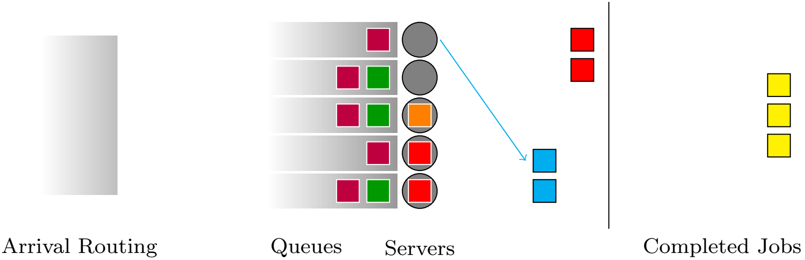

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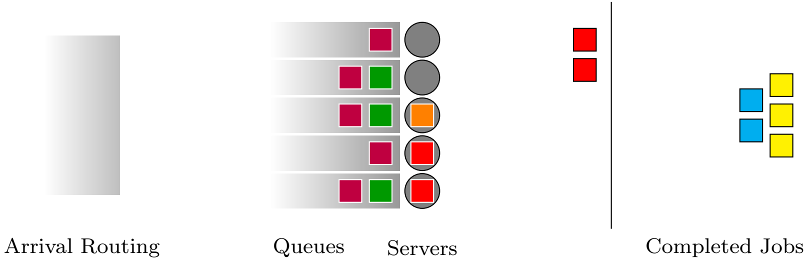

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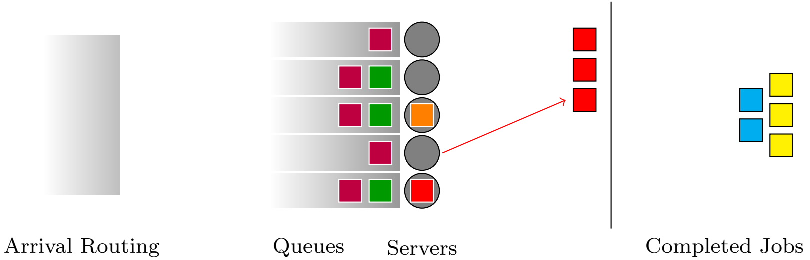

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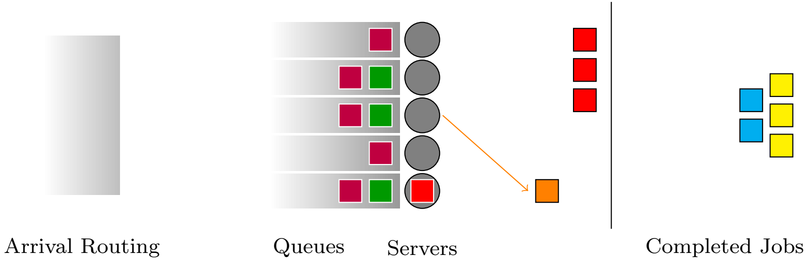

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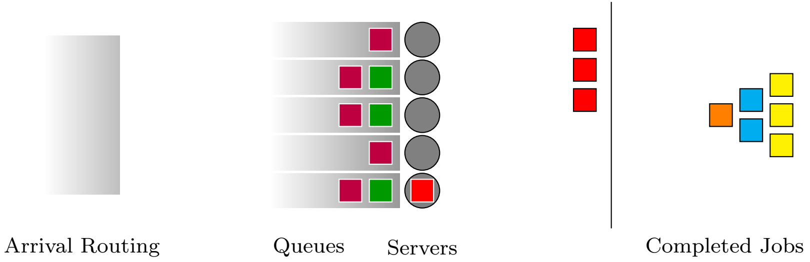

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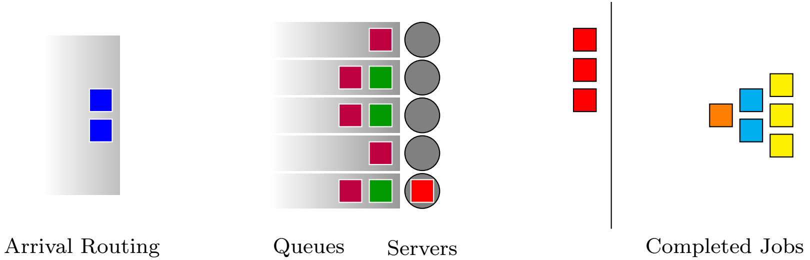

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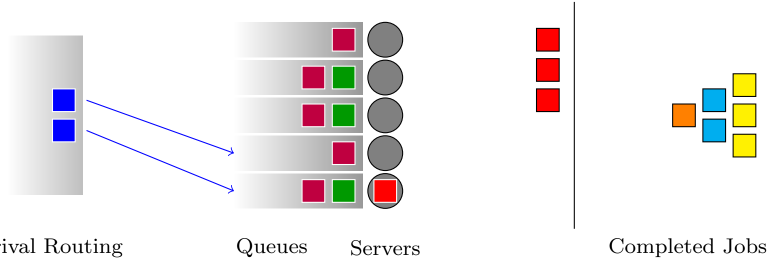

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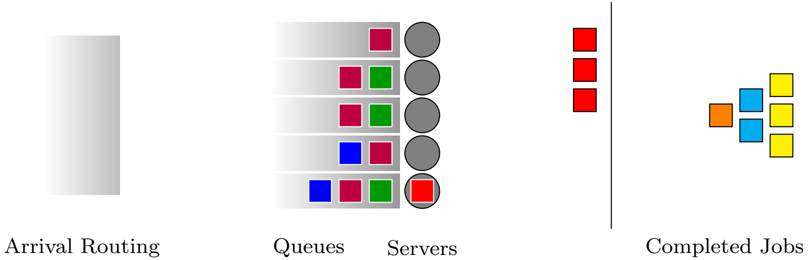

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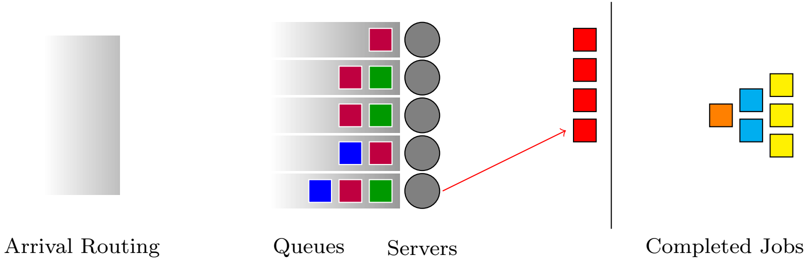

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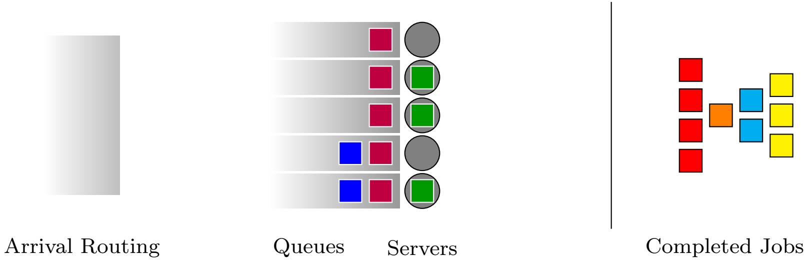

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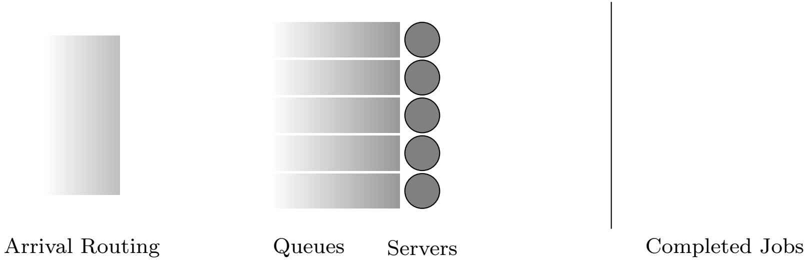

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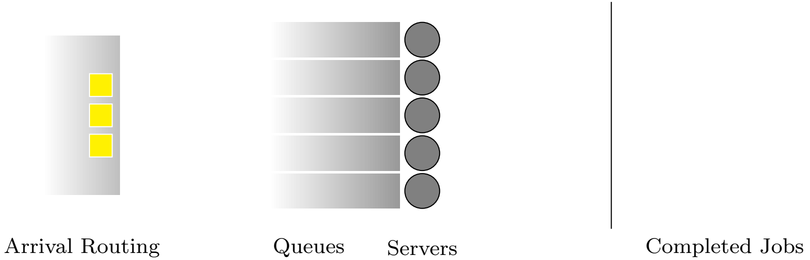

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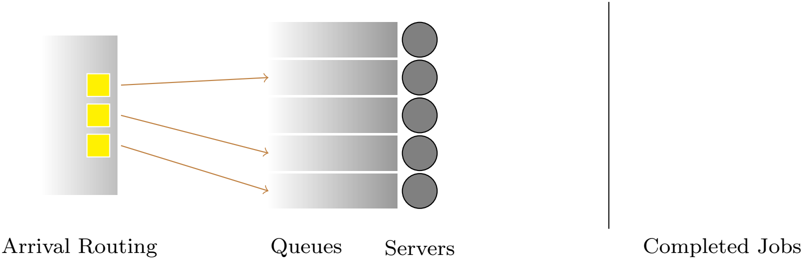

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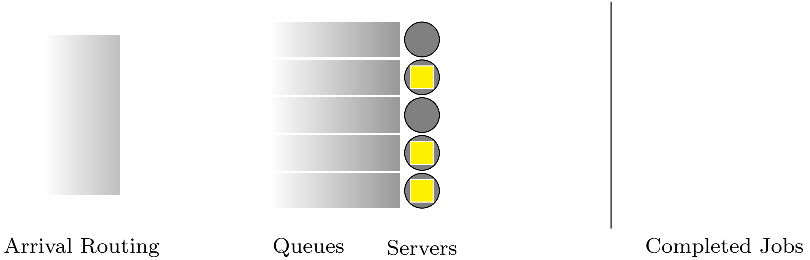

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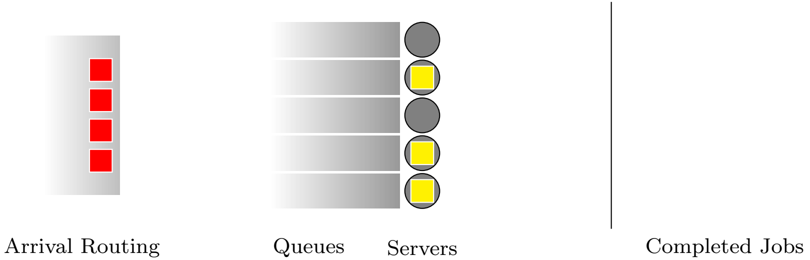

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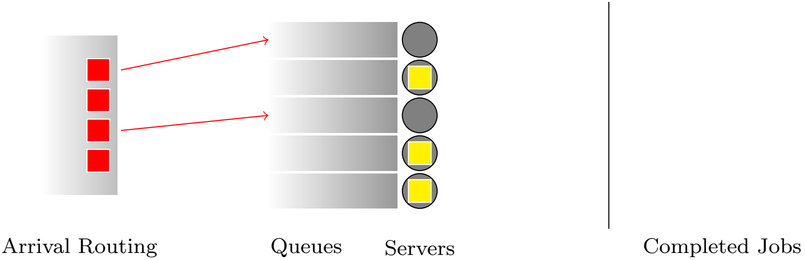

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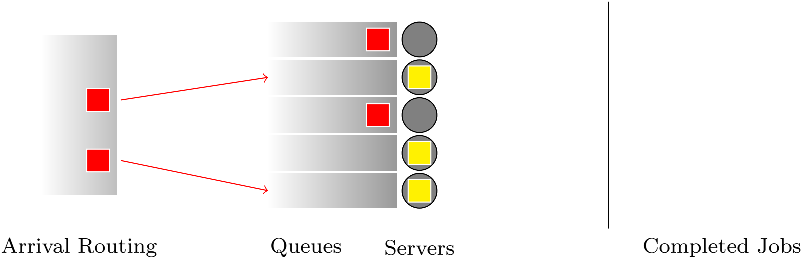

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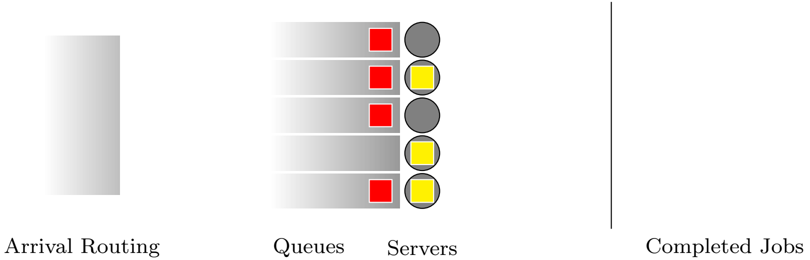

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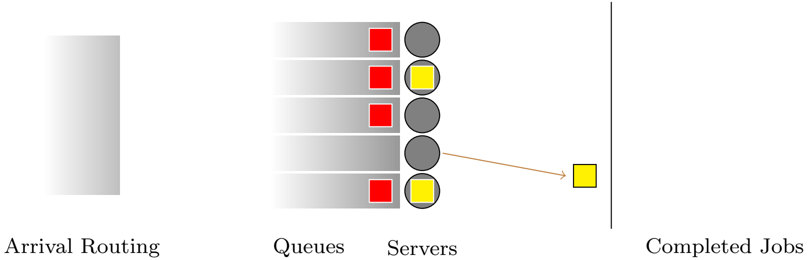

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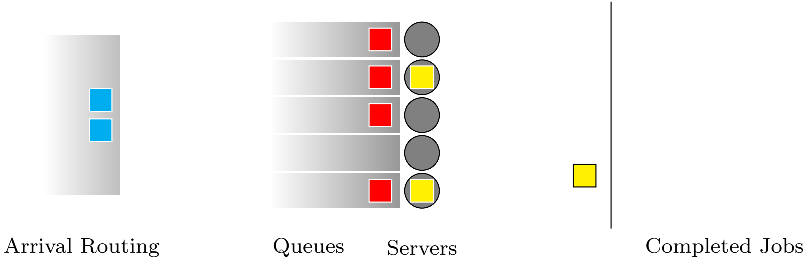

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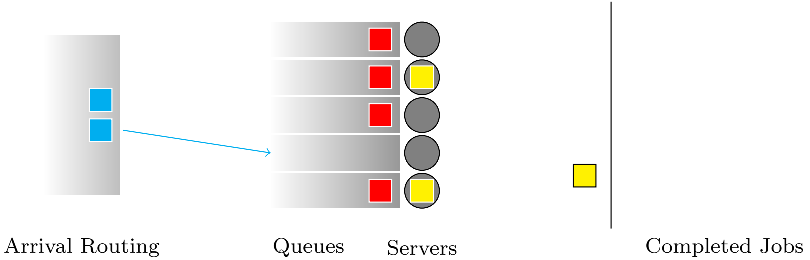

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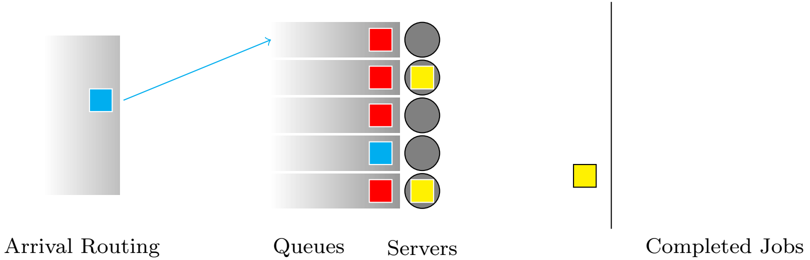

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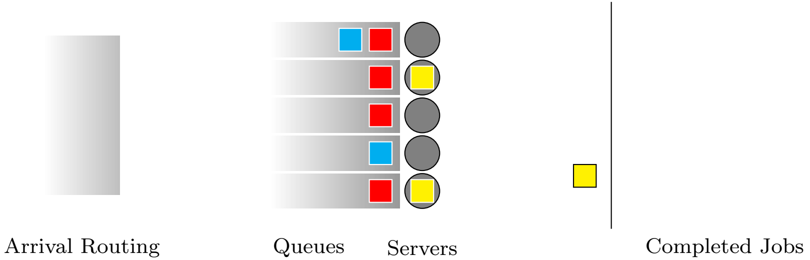

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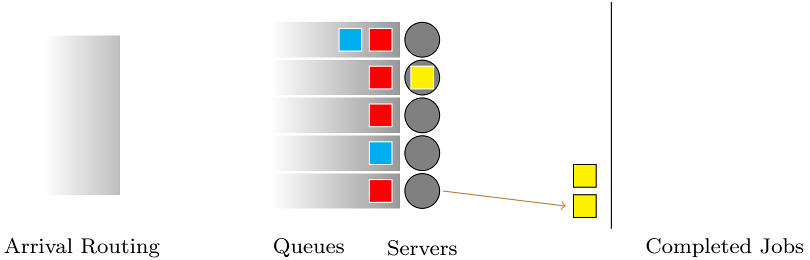

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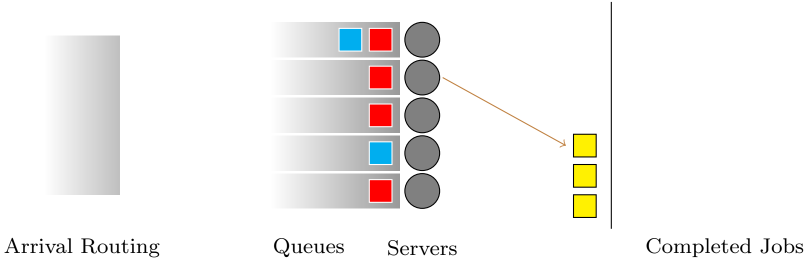

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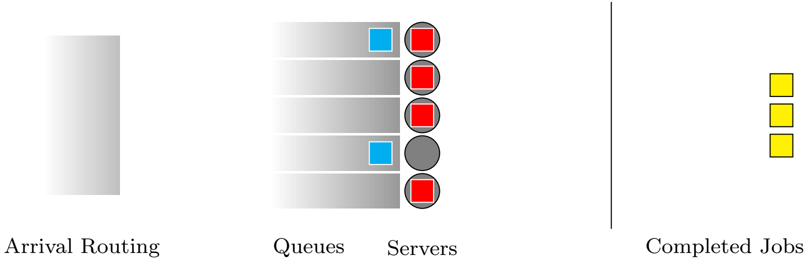

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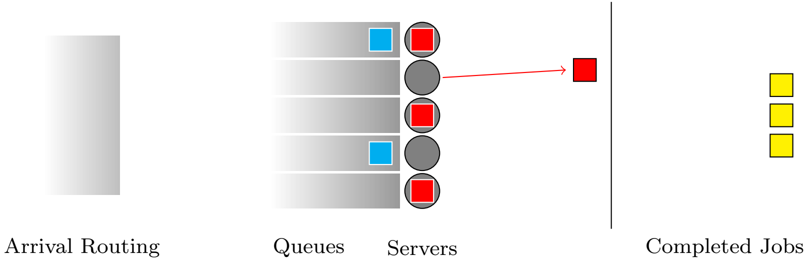

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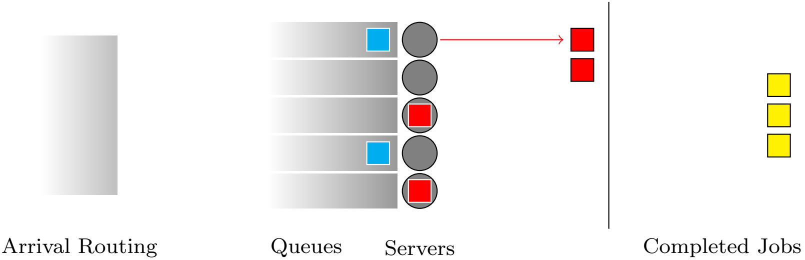

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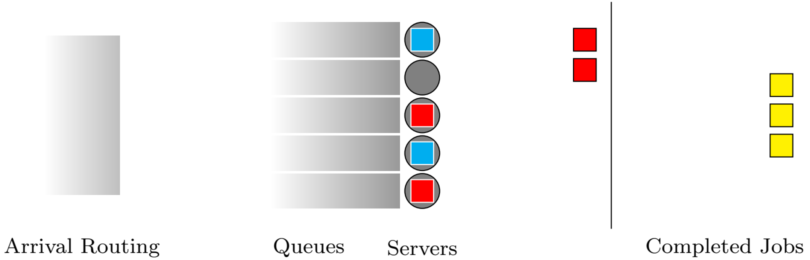

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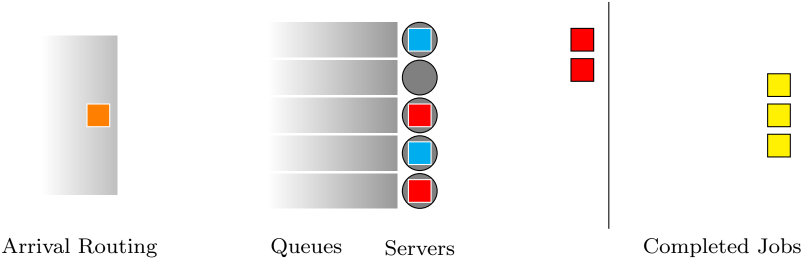

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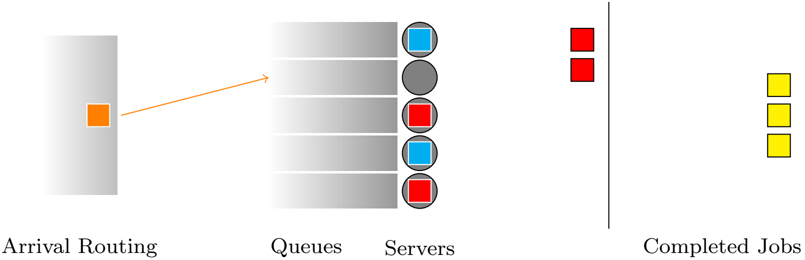

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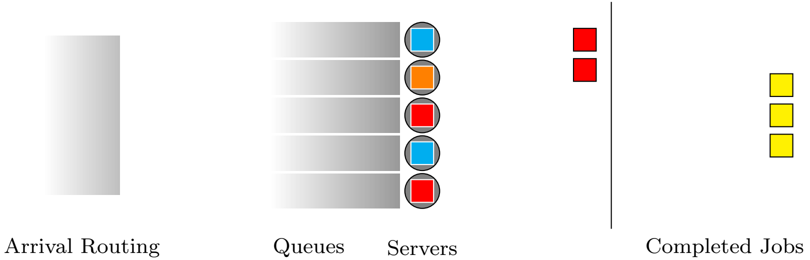

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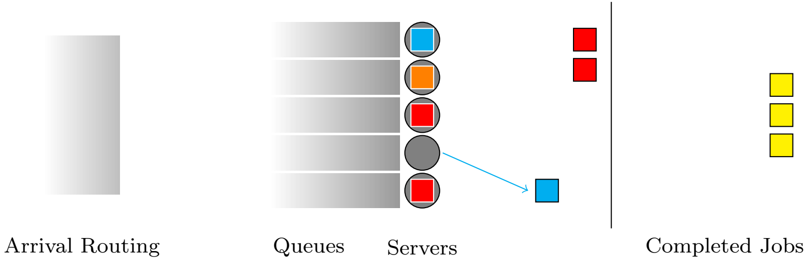

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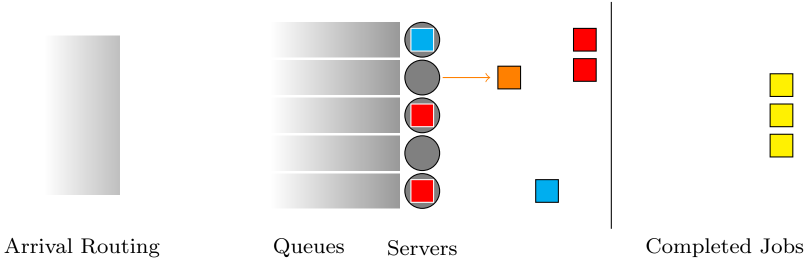

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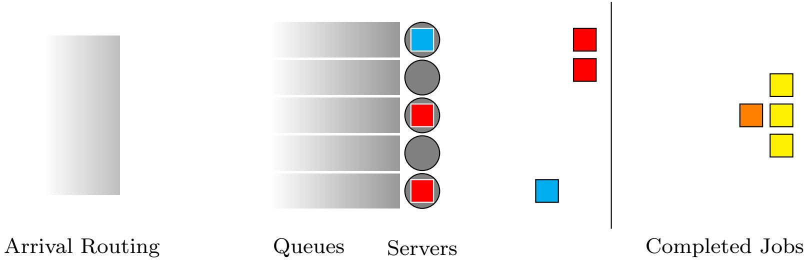

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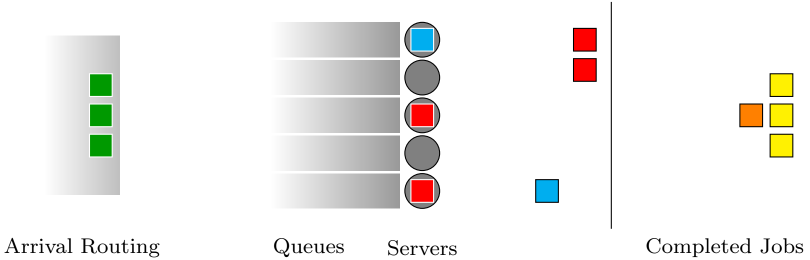

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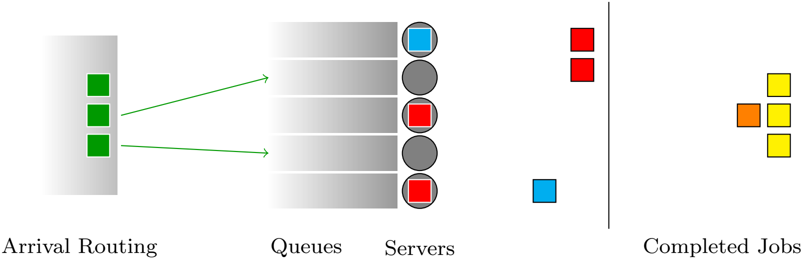

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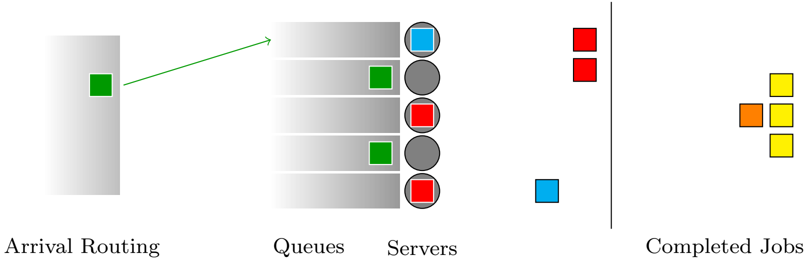

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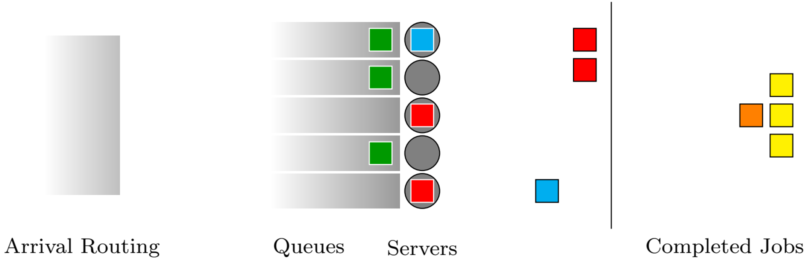

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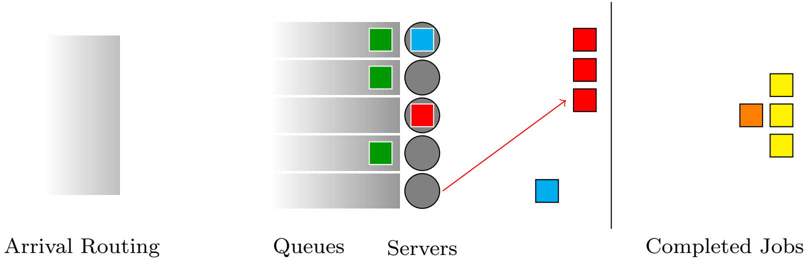

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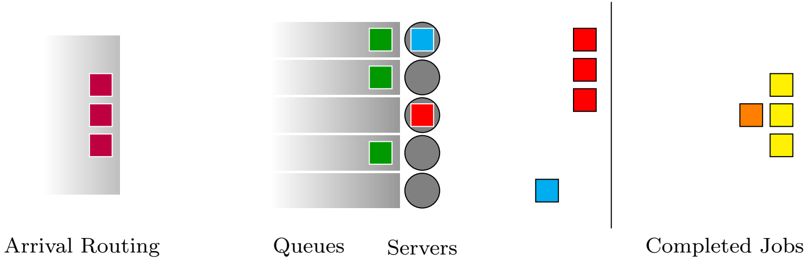

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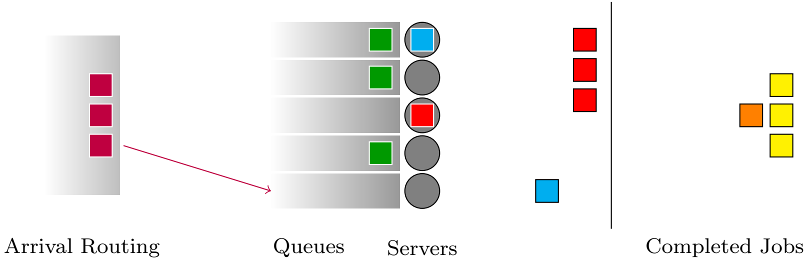

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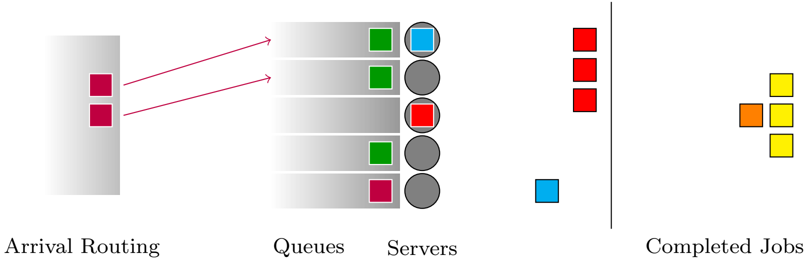

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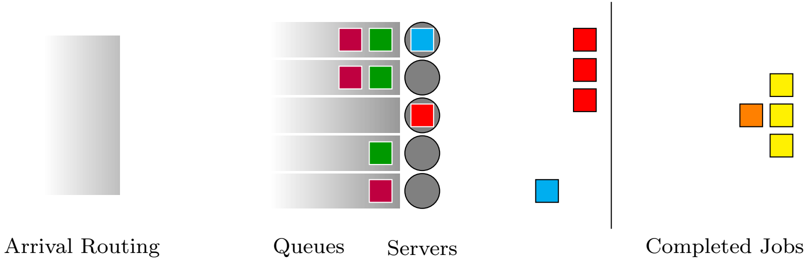

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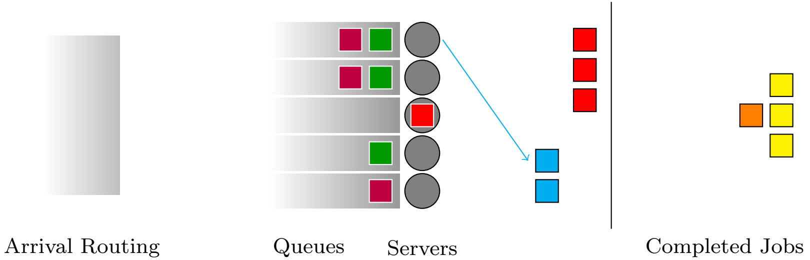

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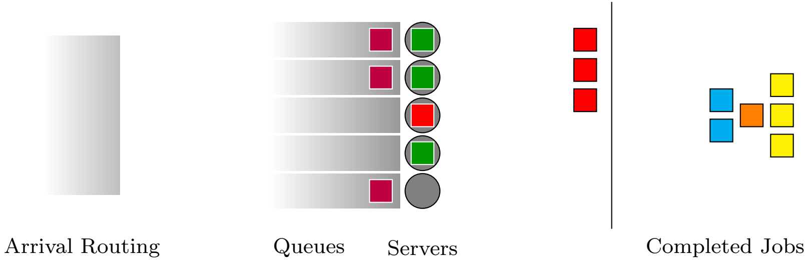

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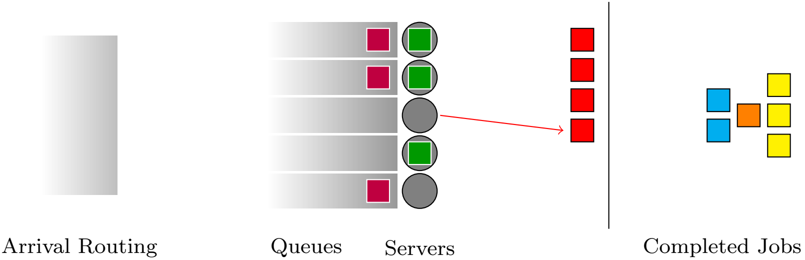

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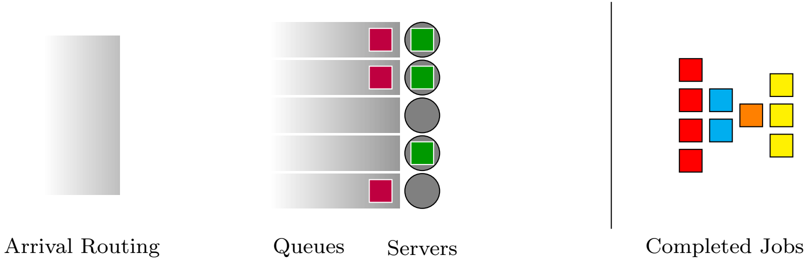

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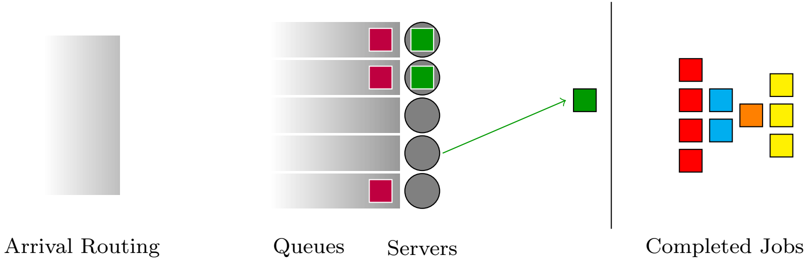

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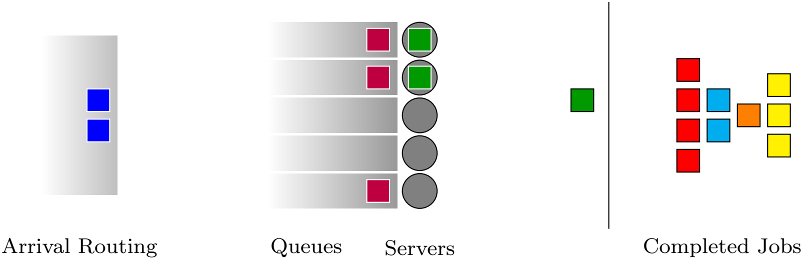

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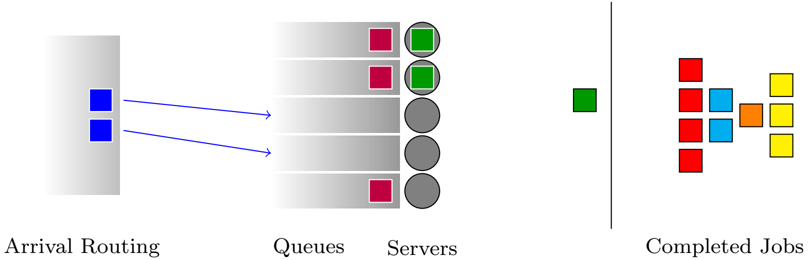

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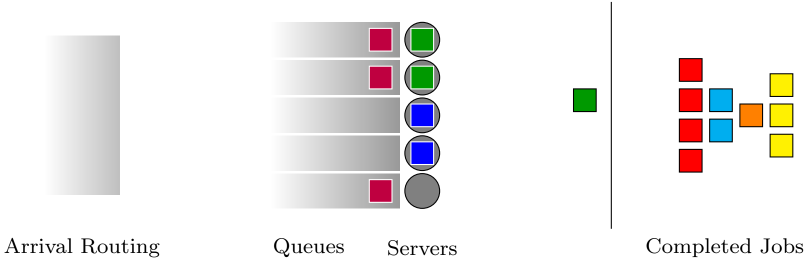

Plan

1 Motivation

2 Model

3 Preliminary Results

4 Numerical Results

5 Conclusions

6 References

Preliminary Results

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- Green [G80]: if $\lambda \mathsf{E}[N]/\mu < 1$, then $W^{(n)} \overset{n}{\Rightarrow} 0$.
- Olvera-Cravioto & Ruiz-Lacedelli [OR14]: $W^{(n)} \overset{n}{\Rightarrow} W$, if there exists $\beta > 0$ such that

$$\mathsf{E}\left[\sum_{i=1}^{N} e^{\beta(\chi_i - \tau_i)}\right] = \frac{\lambda \mathsf{E}[N]^2}{\beta + \lambda \mathsf{E}[N]} \mathsf{E}\left[e^{\beta \chi_1}\right] < 1,$$

where $N \sim f$, $\chi_i \sim S$ and $\tau_i \sim \mathrm{Exp}(\lambda \mathsf{E}[N])$ are independent.

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Numerical Results

We consider:

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  - $N \sim$ Poisson (light tail)
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Numerical Results

We consider:

- 50 – 1,000 servers
- 30,000 jobs
- Distribution of the number of pieces:
  - $N \sim$ Poisson (light tail)
  - $N \sim$ Poisson composed with Pareto (heavy tail)
- Service time of the pieces:
  - $S \sim \text{Unif}(0, 1)$
  - $S \sim \text{Exp}(1)$

Numerical Results: Example 1

Parameters:

$$\lambda = 0.1 \qquad S \sim \text{Unif}(0,1) \qquad \mathsf{E}[N] = 2 \text{ (light tail)}$$

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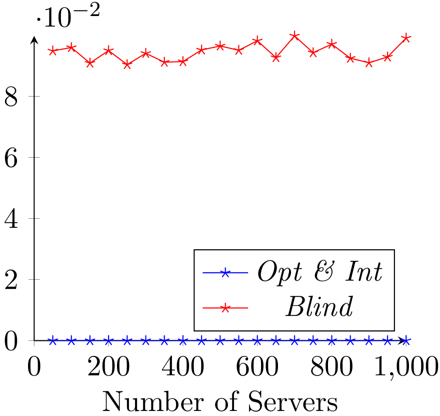
Average Waiting Time:



Numerical Results: Example 2

Parameters:

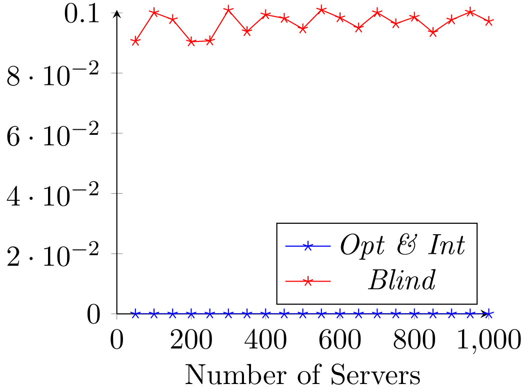
$$\lambda = 0.1 \qquad S \sim \text{Unif}(0,1) \qquad \mathsf{E}\left[N\right] = 2 \text{ (heavy tail)}$$

Numerical Results: Example 2

Parameters:

$$\lambda = 0.1 \qquad S \sim \text{Unif}(0,1) \qquad \mathsf{E}[N] = 2 \text{ (heavy tail)}$$

Average Waiting Time:



Numerical Results: Example 3

Parameters:

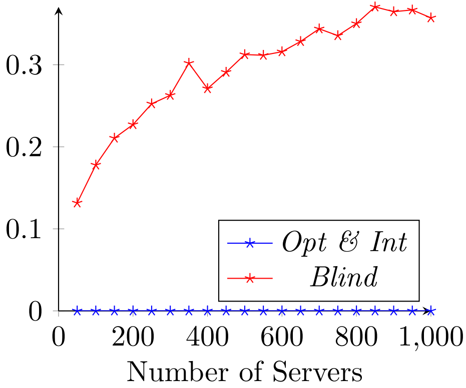
$$\lambda = 0.1 \qquad S \sim \text{Unif}(0,1) \qquad \mathsf{E}\left[N\right] = 20 \text{ (heavy tail)}$$

Numerical Results: Example 3

Parameters:

$$\lambda = 0.1 \qquad S \sim \text{Unif}(0,1) \qquad \mathsf{E}\left[N\right] = 20 \text{ (heavy tail)}$$

Average Waiting Time:



Numerical Results: Example 4

Parameters:

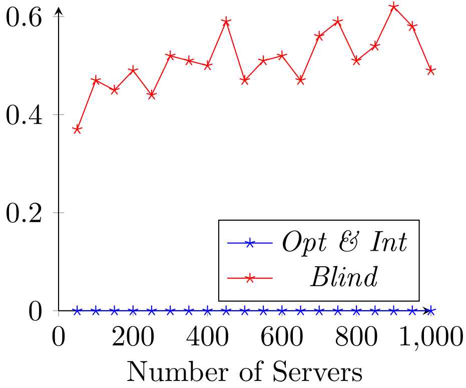
$$\lambda = 0.015 \qquad S \sim \text{Exp}(1) \qquad \mathsf{E}\left[N\right] = 4 \text{ (light tail)}$$

Numerical Results: Example 4

Parameters:

$$\lambda = 0.015 \qquad S \sim \text{Exp}(1) \qquad \mathsf{E}[N] = 4 \text{ (light tail)}$$

Average Waiting Time:



Numerical Results: Example 5

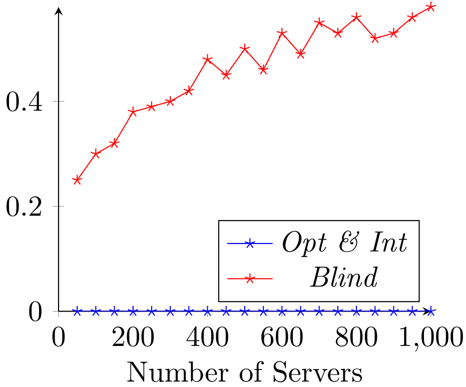
Parameters:

$$\lambda = 0.0025 \qquad S \sim \text{Exp}(1) \qquad \mathsf{E}[N] = 10 \text{ (heavy tail)}$$

Parameters:

$$\lambda = 0.0025 \qquad S \sim \text{Exp}(1) \qquad \mathsf{E}[N] = 10 \text{ (heavy tail)}$$

Average Waiting Time:



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Conclusions

According to our numerical results:

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- Light and heavy tails behave similarly

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According to our numerical results:

- Light and heavy tails behave similarly
- Only knowing which servers are empty, decreases the waiting time significantly

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References

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[OR14] Olvera-Cravioto, Mariana & Ruiz-Lacedelli, Octavio. "Parallel queues with synchronization." (2014) arXiv:1501.00186.

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